Measuring the effect of quantitative easing on the lower bound of interest rate and economic activity ☆

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Abstract

Using the shadow rate affine arbitrage-free Nelson-Siegel models and the shadow rates implied by the consumption-Euler equation, we examine the effect of a quantitative easing policy on the economy in a model with time-varying lower bounds of interest rates. We find evidence of the following: (i) commitment policy in the midst of the 2000s and large-scale asset purchasing after the global financial crisis (GFC) reduce the lower bounds of interest rates; (ii) expanding the monetary base is more influential on the economy than expanding central bank reserves; and (iii) neither of these simple instruments is effective in lowering the lower bounds of interest rates. We argue that a central bank is able to lower the lower bounds, but its effect on the economy is too small.

Keywords: monetary policy, shadow rate, quantitative easing monetary policy, zero interest rate, term structure

JEL classification: E52; E58; E43; E44; G12

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1. Introduction

1. Introduction Recently, several central banks, i.e., the European Central Bank (ECB), Denmark Nationalbank, Sveriges Riksbank, Swiss National Bank, and Bank of Japan (BOJ), adopted a monetary policy of negative interest rates. For example, BOJ announced that the interest rate on current accounts would be -0.1% starting in January 2016, the deposit rate of the Swiss National Bank was set at -0.75% at the end of 2016, and the ECB lowered the interest rate to -0.4%. Before these changes to the monetary policy, people had assumed that a zero interest rate was the lower bound for interest rates. This bounds as called the zero lower bound (ZLB) by academics. However, we now know that the lower bound of interest rates can be negative if the central bank adopts such a monetary policy. Several studies have challenged the issue of ZLB using the notion of Black's shadow rate (Black 1995). Krippner (2013) estimates the currency-adjusted-bond (CAB)-Vasicek model. Christensen and Rudebusch (2015) estimate a three-factor shadow-rate model of Japanese yields. Wu and Xia (2016) consider a multifactor shadow rate term structure model (SRTSM). Ichiue and Ueno (2015) estimate a Black's model with Japa's data and showed that the shadow rate fell into a negative range even when the call rate was approximately 0.5% and before the BOJ adapted a zero interest rate policy in 1999. According to Black (1995), the observed nominal short rate is nonnegative because currency produces a nominal interest rate of zero. In other words, currency imposes a ZLB on yields. Black postulated a shadow short rate, s_t and assumed that the observed instantaneous risk-free rate r_t is given by the

by the greater of the shadow rate or zero, that is, $r_t = \max\{s_t, 0\}$. Following Black (1995), a few studies extended the notion of a zero lower bound to non-zero lower bound. Although Wu and Xia (2016) simply set the lower bound at 25 bp, Kim and Priebsch (2013) treat the lower bound as a free parameter to be estimated, and obtain a value of 14 bp using U.S. Treasury yields. The aim of this study is to measure the effect of quantitative easing in a model with time-varying lower bounds of interest rates during the long period of near-zero interest rates in Japan. In our analysis, we suppose that lower bounds on interest rates change over time, and the central bank can influence these bounds by quantitative easing. More specifically, we assume that a nominal rate, r_t is given by the greater of the shadow rate, s_t and the lower bound z_t, that is , $r_t = \max\{s_t, z_t\}$. In particular, we do not assume that z_t must be nonnegative. We raise several questions that we attempt to answer as follows: What are the economic implications of negative interest rates? Are lower bounds of interest rates affected by a central bank's monetary policy, in particular quantitative easing policy? Should a central bank target either the monetary base or reserves to stimulate the economy through quantitative easing? What are the different effects of a policy of monetary base targeting versus a policy of changing reserves? Similar to our paper, Wu and Xia (2016) argue that the Federal Reserve succeeded in lowering the unemployment rate, using a multi-factor shadow rate term structure model[1]. Our approach is different from Wu and Xia's (2016) in several

[1]Wu and Xia (2017) investigate the effectiveness of the ECB's negative interest rate policy on the yield curve with a new shadow-rate term structure model. They find a 10 bp drop in the lower bound lowers the short-term rate by the same amount, and lowers the 10-year yield by 6 to 8 bp.

respects. With respect to methodology, we estimate the time-varying lower bounds of interest rates and examine the effects of quantitative easing on economic indicators in a model that explicitly incorporates such lower bounds[2]. We also use the shadow rates implied by consumption-Euler equations to estimate the lower bounds. Our critical assumption is that households' consumption satisfies the Euler's equations with respect to shadow rates. Our study is the first to report time-varying lower bounds of interest rates for Japanese data. Using the estimated time-varying lower bounds, our study contributes to the literature that investigates the effectiveness of monetary policy under a near-zero interest rate environment. We find evidence that a commitment policy in the midst of the 2000s and large-scale asset purchasing after the global financial crisis (GFC) lowered the lower bounds of interest rates. We also find new evidence showing that expanding the monetary base is more influential on the economy than expanding the central bank's reserves. However, neither of these simple instruments is effective in lowering the lower bounds of interest rates. We argue that quantitative easing can reduce these lower bounds, but its effect on the economy is too small. This study also contributes to the econometric methodology for estimating the effect of the monetary policy of quantitative easing. Compared to the existing literature, our method is novel in that we use the shadow rates implied by the consumption-Euler equation. The method consists of four parts. First, we derive Euler equations for two types of utility functions, constant relative risk aversion (CRRA) type and habit type, and obtain the implied

[2] Christensen and Rudebusch (2015, p238-239) state that they are skeptical about the use of a non-zero lower bound, partly because of the lack of motivation.

real and nominal rate of interest rates. Second, we treat the implied rates as shadow rates and estimate the shadow rate affine arbitrage-free Nelson-Siegel (AFNS) models by the Kalman filter. The AFNS model belongs to a class of broader affine term structure models (Duffie and Kan 1996, Dai and Singleton2000, Duffee 2002). Third, using the estimated parameters of the AFNS models, we compute the time-varying lower bounds of the actual interest rates, as the theoretical option price. Finally, we estimate the factor-augmented vector autoregressive(FAVAR) model including a lower bound as one of the variables. The final analysis provides impulse responses of the macroeconomic indicators to the quantitative easing, i.e., the expansion of the monetary base and the BOJ's reserves. The paper is organized as follows. Section 2 describes our methodology. Section 3 provides the estimated results. Section 4 concludes the paper.

2. Econometric methodology

2.1. The outline of econometric methodology

Our econometric approach consists of the following four steps.

Step 1 : We derive the implied real rates from the consumption-Euler equation.

Step 2 : From the series of implied real rates and inflation rates, we calculate the nominal shadow rates.

Step 3 : We estimate shadow-rate AFNS models, using the estimated shadow rates. We derive implied lower bounds using the estimated parameters of AFNS models by numerically solving the theoretical option pricing formula.

Step 4 : We estimate the FAVAR model, including implied lower bounds and a quantitative easing measure.

Here after, we explain these four steps in turn.

2.2. Euler equation and the implied real rate

In Step 1, we consider Euler equations for two utility functions. First, we consider the usual CRRA utility

$$u(C_t) = \frac{1}{1-\alpha} C_t^{1-\alpha} \tag{1}$$

where C_t denotes consumption at time t and α denotes relative risk aversion. For an individual maximizing his or her discounted lifetime utility, the consumption-Euler equation of consumptions between t and $t+T$ becomes

$$\exp(-r(t,T)T) = \beta(t,T) E_t \left[\left(\frac{C_{t+T}}{C_t} \right)^{-\alpha} \right] \tag{2}$$

where $\beta(T)$ is a discount factor, $r(t,T)$ is a real shadow rate at time t maturing at time $t+T$, and E_t denotes the conditional expectation at time t.

Defining $c_t = \ln C_t$ and assuming a log-normal distribution, the above equation becomes

$$\exp(-r(t,T)T) = \beta(t,T) \exp\big(-\alpha(E_t(c_{t+T}) - c_t) + 0.5\alpha^2 V_t(c_{t+T})\big) \tag{3}$$

where V_t denotes the conditional variance at time t.

Second, we consider the habit model of Fuhrer (2000), which assumes that the consumer's period utility function is

$$u(C_t, H_t) = \frac{1}{1-\alpha}\left(\frac{C_t}{H_t^{\gamma}}\right)^{1-\alpha} \tag{4}$$

where H_t is the habit level of consumption at period t and γ is a parameter indexing the importance of habit. When the autocorrelation coefficient of H_t is close to zero, the utility function becomes

$$u(C_t, H_t) = \frac{1}{1-\alpha}\left(\frac{C_t}{C_{t-1}^{\gamma}}\right)^{1-\alpha} \tag{5}$$

Then, the Euler equation is given by

$$\beta(t,T)\exp(-r(t,T)T) = \frac{\exp(a_t) - \beta(t,T)\gamma\exp(b_t)}{\exp(d_t) - \beta(t,T)\gamma\exp(e_t)} \tag{6}$$

where

$$\begin{aligned}
a_t &= \gamma(\alpha-1)c_{t-1} - \alpha c_t \\
b_t &= (\gamma(\alpha-1)-1)c_t + (1-\alpha)E_t c_{t+T} + 0.5(1-\alpha)^2 V_t(c_{t+T}) \\
d_t &= \gamma(\alpha-1)c_t - \alpha E_t c_{t+T} - E_t\pi_{t+T} + 0.5\alpha^2 V_t(c_{t+T}) + 0.5V_t(\pi_{t+T}) \\
&\quad + \alpha\mathrm{cov}_t(c_{t+T}, \pi_{t+T}) \\
e_t &= (\gamma(\alpha-1)-1)E_t c_{t+T} + (1-\alpha)E_t c_{t+T+1} - E_t\pi_{t+T} + 0.5(\gamma(\alpha-1)-1)^2 V_t(c_{t+T}) \\
&\quad + 0.5(1-\alpha-1)^2 V_t(c_{t+T+1}) + 0.5V_t(\pi_{t+T}) + (1-\alpha)(\gamma(\alpha-1)-1)\mathrm{cov}_t(c_{t+T}, c_{t+T+1}) \\
&\quad - (1-\alpha)(\mathrm{cov}_t(\pi_{t+T}, c_{t+T+1}) + \mathrm{cov}_t(\pi_{t+T}, c_{t+T}))
\end{aligned}$$

Following Canzoneri et al. (2007), we derive the implied real rates $\hat{r}(t,T)$

for each of the two utility functions using Eqs. (3) and (6), respectively.

2.3. Nominal shadow rate implied by Euler equation In Step 2, we calculate a nominal shadow rate $s(t,T)$ using the implied real rate and the inflation rate $\pi(t,T)$ as

$$\hat{s}(t,T) = \hat{r}(t,T) + \pi(t,T), \tag{7}$$

as the implied by Fisher equation.

2.3. The upper bound as a strike price of shadow bond option

In Step 3, we consider shadow-rate AFNS models. As explained briefly in the introduction, we extend Black's notion of shadow rate as

$$i(t,T) = \max\{s(t,T), z(t,T)\} \tag{8}$$

where $i(t,T)$ is a nominal interest rate and $z(t,T)$ is a lower bound on the nominal interest rate. The lower bound of interest rates corresponds to the upper bound of bond prices. When the ZLB applies, the upper bound of bond prices is equal to its face value If we define the upper bound of a bond price as $Z(t,T)$, the bond price cannot exceed the upper bound $Z(t,T)$. This means that a bondholder must sell the shadow bond at price $Z(t,T)$ when this occurs. In other words, the bond holder is a seller of a call option on a shadow bond with a strike price $Z(t,T)$. Therefore, holding a bond with an upper bound is the same as holding the shadow bond and selling the call option. The upper bound of the bond price $Z(t,T)$ can be greater than, equal to, or smaller than the bond ' s face value. If holding cash involves no friction, and cash pays zero interest rate, the upper bound is just the bond's face value. If there are relative benefits to holding bonds or relative costs to holding cash, and if cash pays zero interest rate, the upper bound becomes greater than the bond's face value because the positive cost of cash means that the actual rate on holding cash, including this cost, is negative. In contrast, if there are relative benefits to holding cash or relative costs to holding bonds, and if cash pays a zero interest rate, the upper bound becomes lower than the bond's face value because the positive benefit of holding cash means that the actual rate on holding cash, including this benefit, is positive. The central bank's reserve policy affects the benefit/cost ratio of holding cash, as long as we consider reserves equivalent to cash. The Federal Reserve began to pay interest on reserves in 2008 (Emergency Economic Stabilization Act). Following FRB, BOJ also began to pay interest on the excess reserves in 2008. Curdia and Woodford (2011) argue that the optimal interest payment on reserves should be equal to the policy rate. When the interest rate on reserves is positive, this interest rate can be regarded as a relative benefit of holding reserves (or cash), which makes the lower bound of the interest rate positive. When the interest rate on reserves is negative, this interest rate can be regarded as the relative cost of holding reserve (or cash), which makes the lower bound of the interest rate negative. In this way, the central bank can influence the lower bound of the interest rate by changing the benefit/cost ration of holding cash relative to bonds. An alternative way to affect the benefit/cost ratio is to buy a large amount of bonds through open market operations. Several studies have investigated the effect of large-scale asset purchases (Gertler and Karadi 2013, D'Amico and King 2013, D'Amico et al. 2012, Chen et al.

2012, Gagnon et al. 2011, Vayanos and Vila 2009). For example, the Federal Reserve purchased \$1.75 trillion in 2009, the Bank of England purchased \$275 billion in 2012, and BOJ purchased ¥100 trillion as of 2012. Large-scale asset purchase programs, which typically leave a low amount of bonds in the market, increase the cost of holding bonds relative to cash because financial institutions must hold bonds for regulatory reasons such as Basel regulation or for allocating their assets efficiently by holding an appropriate amount of risk-free assets. Such policy makes it difficult for financial institutions to find sellers of bonds, which means that the benefit of holding bonds related to regulatory or other reasons increases. As mentioned in the introduction, we do not observe the lower bounds of interest rates, which can be positive, zero, or negative. Hence, we need to estimate these lower bounds by partly applying the method of Krippner (2012) and Christensen and Rudebusch (2015) (hereafter abbreviated as KCR) [3]. The idea of this method comes from Black's option theory of interest rates. We utilize the fact that holding a bond with an upper bound is the same as holding the shadow bond and selling a call option on the bond. As has been argued by KCR, the final value to the bondholder, W_t, is given by the smaller of either the shadow bond price $S(T, T)$ or the upper bound price $Z(T, T)$. That is,

$$W_T = \min\{S(T,T), Z(T,T)\} = S(T,T) - \max\{S(T,T) - Z(T,T), 0\} \quad (9)$$

[3] See also Christensen and Rudebusch (2016).

The last term on the right-hand side represents the payoff of a call option with a strike price of $Z(T,T)$ and an original asset $S(T,T)$ Therefore, holding a bond with an upper bound is equivalent to holding the shadow bond and selling this call option. Following KCR, we consider a European call option at time 0 with maturity T and a strike price $Z(T,T)$ written on the shadow discount bond maturing at $T+\delta$. $T+\delta$ is the shortest maturity available after time T. This call option approximates the option in Eq. (9), whose value is defined as

$$\begin{aligned} &C(T,T+\delta,Z(T,T)) = E_0^Q\left[S(0,T)\max\{S(T,T+\delta)-Z(T,T),0\}\right] \\ &= S(0,T+\delta)N(d_1) - Z(T,T)S(0,T)N(d_2) \end{aligned} \tag{10}$$

where

$$\begin{aligned} d_1 &= \left(\ln\frac{S(0,T+\delta)}{S(0,T)Z(T,T)} + 0.5T\nu_t^2\right)\frac{1}{\nu_t\sqrt{T}}, \\ d_2 &= d_1 - \nu_t\sqrt{T}, \end{aligned} \tag{11}$$

ν_t is the diffusion coefficient, and N is the cdf of the standard normal distribution. The conditional expectation in Eq. (10) is taken with respect to the risk neutral measure Q. Then, the value of the auxiliary bond with an upper bound $Z(T,T)$ is approximately equal to

$$P(T,T+\delta) = S(0,T+\delta) - C(T,T+\delta,Z(T,T)) \tag{12}$$

, which comes from Eq. (9). As already mentioned, the value of this auxiliary bond is the value of the shadow bond minus the price of a call option.

Following KCR, we consider the instantaneous forward rate on the auxiliary bond, which is defined as

$$\underline{f}(t,T) = \lim_{\delta \to 0} \left[-\frac{d}{d\delta} P(T, T+\delta) \right] \tag{13}$$

where we assume that the actual bond is approximated by the auxiliary bond and regard $\underline{f}(t,T)$ as the forward rate of the actual bond[4]. Furthermore, following KCR, the forward rate on the shadow bond $f(t,T)$ and that of the actual bond $f(t,T)$ satisfy

$$\underline{f}(t,T) = f(t,T) + g(t,T). \tag{14}$$

where the adjustment term $g(t,T)$ is given by[5]

$$\begin{aligned} g(t,T) &= \lim_{\delta \to 0} \left(\frac{d}{d\delta} \left(\frac{C(t,T,T+\delta, Z(t,T))}{P(t,T)} \right) \right) \\ &= z(t,T) + (f(t,T) - z(t,T)) N \left(\frac{f(t,T) - z(t,T)}{\nu_t} \right) \\ &\quad + \nu_t \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{1}{2} \left[\frac{f(t,T) - z(t,T)}{\nu_t} \right]^2 \right) \end{aligned} \tag{15}$$

[4]See Eq. (5) in Christensen and Rudebusch (2015). KCR's currency-adjusted-bond-Vasicek model consider the bond price at time t maturing at the shortest maturity available, $t+\delta$. Investors can either buy the bond at a price $P(t, t+\delta)$ and receive one unit of currency at the maturity date or just hold the currency. The availability of currency implies that the last incremental forward rate of any bond will be nonnegative due to the future availability of currency in the immediate time prior to its maturity. When letting $\delta \to 0$, the continuous limit identifies the nonnegative instantaneous forward rate.

[5]This equation is similar to the last equation on page 238 of Christensen and Rudebusch (2015).

The adjustment term $g(t,T)$ represents the change in the value of call option maturing at time T.

2.4. The shadow-rate AFNS model

We consider a three-factor AFNS model as in Christensen and Rudebusch (2015) and Christensen et al. (2011). The shadow rate is assumed to follow

$$s(_t, T) = X_t^1 + X_t^2, \tag{16}$$

and the three state variables follow

$$\begin{pmatrix} dX_t^1 \\ dX_t^2 \\ dX_t^3 \end{pmatrix} = -\begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda & -\lambda \\ 0 & 0 & \lambda \end{pmatrix} \begin{pmatrix} X_t^1 \\ X_t^2 \\ X_t^3 \end{pmatrix} dt + \begin{pmatrix} \sigma_{11} & 0 & 0 \\ \sigma_{21} & \sigma_{22} & 0 \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} \begin{pmatrix} dW_t^{1,Q} \\ dW_t^{2,Q} \\ dW_t^{3,Q} \end{pmatrix} \tag{17}$$

As is well known, the yield of AFNS(3) is given by

$$\begin{aligned} iy(t,T) = X_t^1 &+ \left(\frac{1-e^{-\lambda(T-t)}}{\lambda(T-t)}\right) X_t^2 \\ &+ \left(\frac{1-e^{-\lambda(T-t)}}{\lambda(T-t)} - e^{-\lambda(T-t)}\right) X_t^3 - \frac{A(t,T)}{T-t} \end{aligned} \tag{18}$$

, where the factor loading structure is similar to that in Nelson and Siegel (1987). [6]. The three factors X_t^1, X_t^2, and X_t^3 represent as the factor loadings of level, slope, and curvature, respectively. The last term on the right hand side $A(t,T)/(T-t)$ is a yield-adjustment term that guarantees no arbitrage. The drift coefficient λ and the volatility matrix in Eq. (17) determines this

[6] See the first equation in page 235 of Christensen and Rudebusch (2015).

adjustment term and the volatility of the yields $\nu_t^2 = \mathrm{V}_t(\ln P(t,\tau))$[7].

2.4.1. *The procedure for estimating the lower bounds*

Now we explain the procedure for estimating the lower bounds of interest rates in Step 3. The procedure consists of the following two parts: (3-1) Estimating the parameters of the AFNS model and (3-2) Estimating the implied lower bounds. In step (3-1), using shadow rates for each maturity given in Eq. (7), we estimate a state-space model consisting of two equations. One is a transition equation that is derived from the discrete version of the state process, given in Eq. (17). The transition equation describes the dynamics of the unobserved state factors. The second equation is a measurement equation given in Eq. (18). We add Gaussian white noise random disturbances to each equation and assume that the disturbances are orthogonal to each other.

We estimate a state-space model using the Kalman filter and obtain σ_{ij}^2 and λ for the AFNS(3) model. Using these estimated parameters, we compute the conditional volatility of yields ν_t.

In step (3-2), using the conditional volatility of yields ν_t, we calculate the lower bound given by Eqs. (14) and (15). Inserting the forward rates on actual bonds and shadow bonds into Eq. (14), we obtain the estimate of lower bound $\hat{z}(t,T)$ in Eq. (15) by solving numerically the nonlinear equation.

[7] The volatility of the yields is given by ν_t in page 236 of Christensen and Rudebusch (2015).

2.5. FAVAR estimation

Finally, in Step 4, using the estimated lower bound $\hat{z}(t,T)$, we consider several FAVAR specifications of the above model. We consider three unobservable factors and one observable variable, which is either the monetary base or the BOJ's reserves. We consider the observational equations consisting of a series of 131 informational variables listed in Appendix Table A1.

We employ a two-step approach as in Bernanke et al.(2005) to estimate the three-factor FAVAR. The procedure consists of two steps. In step (4-1), we estimate principal components of the informational variables X_{it}. In step (4-2), we estimate the usual VAR of three factors F_t that were estimated in step (4-1). and one of the exogenous monetary variables, M_t. We calculate the We calculate the impulse response of major economic indicators to the impulses, following the method of Wu and Xia (2016). The indicators are IP (Industrial Production), CU (Capacity Utilization), UR (Unemployment Rate), HS (Housing Start), PI (Price Index), and LB (Lower Bound). Defining a vector of these indicators as Y_t, the observation equation is specified as

$$Y_t = a + b_F F_t + b_M M_t + \eta_t \tag{19}$$

Letting the monetary shock be ϵ_t, the impulse response of indicator Y_t to ϵ_t at time $t+l$ is given by

$$\sum_{j=0}^{l} \left(\sum_{k=1,2,3} b_{F_k} \frac{\partial F_{t+j,k}}{\partial \epsilon_t} + b_M \frac{\partial M_{t+j}}{\partial \epsilon_t} \right) \tag{20}$$

, where $F_{t+j,k}$ denotes the k-th factor at $t+j$.

3. Estimation results

3.1. Data

Our monthly sample data set starts in January 1991 and ends in December 2016. We employ consumption data from the Family Income and Expenditure Survey provided by the Statistics Bureau of Ministry of Internal Affairs and Communications. We also employ consumer price index (CPI) provided by the same Bureau. We consider the growth rate of consumption in real terms and interest rates for one, three, six, and twelve months. In this paper, we do not analyze maturities longer than one year. This is because the evidence indicates that the lower bound of shorter maturity tends to be negative and to vary over time. This is because evidence indicates that the lower bound of shorter maturity terms tends to be negative and vary over time. We adjust the influence of a rise in consumption tax in April 1997 and April 2014, following the BOJ's report[8]. We consider two CPIs and rates of consumption growth with and without residential expenditure. Since the results turned out to be similar, we only report below the results using data including residential expenditure.

3.2. Calculating shadow rate and lower bound

This section explains the results of steps 1, 2 and 3. Figure 1 and 2 depict the actual nominal rates $i(t,T)$, the implied shadow rates $\hat{s}(t,T)$ calculated

[8]The report's title is *The method of estimating consumer prices adjusted for consumption tax*, which is published by the Research and Statistics Department of BOJ in November 2016.

according to Eqs. (3), (6), and (7), and the lower bound $\hat{z}(t,T)$ calculated according to Eq. (8) for the maturity. The utility function is either a standard CRRA or Fuhrer (2000)'s habit utility. According to our GMM estimation results of the Euler equations, the typical estimated β is 0.998 and the coefficient of relative risk aversion is $\alpha = 0.01$[9]. Our implied shadow rate for each maturity term mostly stays around zero. It occasionally exhibits a lower or higher level than that found in Christensen andRudebusch (2015, Figure 5, B-AFNS(3) model).

For the 2002–2007 period during which near-zero or negative shadow rates are reported in Christensen and Rudebusch (2015), our implied shadow rate exhibits positive rate ranging from 1 bp to 5 bp. Ichiue and Ueno (2015) report an even lower negative - rate, than that reported in Christensen and Rudebusch (2015) during that period[10].

Although Kim and Singleton (2012) also report a negative shadow rate during 2002–2006 period, their reported rate is more closer to that level reported in Christensen and Rudebusch (2015) than that of Ichiue and Ueno (2015). Compared to the rates reported in the existing literature, our implied rates are relatively higher. Ours is the closest to that of the B-AFNS(3) model of Christensen and Rudebusch (2015). One reason that our implied shadow rate is relatively high is that we estimate it using consumption-Euler

[9] Canzoneri et al.(2007) report that the mean of the implied real rate in the U.S. is 7.08% under the assumption of $\alpha = 2$ and $\beta = 0.993$. Such a high relative risk aversion coefficient produces a high implied real rate, which is inconsistent with recent Japanese data.

[10] Ichiue and Ueno (2015) estimate a -3% of shadow rate around the 2002–2003 period at minimum. Gorovoi and Linetsky (2004) also report that the implied value of the shadow rate was negative.

equation, in contrast to other studies. For data on the U.S., there are a few studies estimating lower bounds of interest rates. Although Wu and Xia (2016) set the lower bound 25 bp in their main analysis, they also estimate it using a robustness check. The estimated time-invariant lower bound is 19bp in the U.S, which is close to the estimated lower bound found in Kim and Priebsch (2013). However, there is no literature reporting the lower bound for Japan. As shown in Figure 1 and 2, each dynamic of lower bound seems to show similar actuations, independent of the utility function and the maturity term. Except for the period from March 2001 to July 2006, and for the period after the GFC, the lower bounds of interest rates nearly coincide with the actual rates, which implies that strike prices of call options were far lower than the actual price of the bond. Hence, the prices of call options were very low. In other words, there was room for lowering the shadow rate below the level implemented by the BOJ. The result suggests that the BOJ kept interest rates higher than the corresponding lower bounds. In contrast, there are large gaps between the lower bound and the rate in the two periods of mid 2000s and after GFC. The gaps seem to increase with maturity term. For example, the gap is 0.1% in the mid 2000s for one month and 0.4% for six months. The lower bounds of interest rates for shorter maturity terms tend to be higher than those of longer maturity terms. This tendency is qualitatively inconsistent with the observation that zero is the appropriate lower bound with the lowest recorded one- and two-year yield being 0.0 bp and 1.3 bp, respectively, whereas the six-month yield broke the zero bound on a few occasions but was never lower than −2 bp according to Christensen and Rudebusch (2015, p238-239). The negative effect of the time to maturity on the lower bound suggests that a longer-maturity bond provides more benefits relative to cash than a shorter-maturity bond. However, in the midst of 2000s, the BOJ did not pay the interest rate on reserves nor buy a large amount of Japanese Government Bond (JGB). That was a period of so-called ‘commitment’ (Oda and Ueda2007). In 2001, the BOJ announced that it is committed to a monetary policy of continued quantitative easing until the inflation rate became steadily greater than zero. Such a commitment policy lowered the lower bounds of interest rates in the midst of the 2000s.

Figure 1

Figure 2

In December. 2008, the BOJ announced that it strengthened its commitment to a monetary easing policy. It began to pay interest rates on the excess reserves of 10 bp and started a large-scale purchasing of JGB. It increased the amount of purchasing every year after 2008. The amount of purchasing increased further after the BOJ introduced quantitative and qualitative monetary easing in April 2013. The graphs in Figures 1 and 2 indicate that such large-scale asset purchasing also contributed to lowering the lower bounds. Table 1 summarizes the statistics for the actual rate, the implied shadow rate, and the estimated lower bound. For example, the sample average of the lower bound for one month under the assumption of CRRA utility is 0.226%, and that of the shadow rate is 0.149%. The table confirms the previous observation that the lower bounds of interest rate tend to be negative as the maturity term increases. In addition, the sample average of the shadow rates is positive. The gap, which is defined as the actual rate minus the lower bound, is also positive in every cell.

Table 1

Table 2 reports the parameter values of each AFNS model. λ and σ_{ij} are drift and volatility coefficients under a Q-measure, respectively. κ_{ij}^{P} is the drift coefficient under a P-measure.

Table 2

3.3. The effect of quantitative easing

3.3.1. Impulse response

We now report the results of the analysis on how changes in the quantitative easing policy affect the lower bound of interest rates and the economy. Economists have generally argued that a near-zero interest rate no longer conveys any information regarding a monetary policy stance. One solution to overcome the issue of an unconventional monetary policy is to use a shadow rate. Wu and Xia (2016) argued that a shadow rate model can be used to summarize the macroeconomic effects of an unconventional monetary policy. This study takes a further step to argue that a monetary policy stance affects the lower bound of interest rates and economy, when assuming time-varying lower bounds. Figures 3 to 6 plot the impulse responses of the following six macroeconomic variables: IP, CU, UR, HS, PI, and LB, to the impulse, namely, quantitative easing measure M_t (monetary base or BOJ reserves). The impulse is set at 1% change of the standardized series of quantitative easing measure.

Figure 3

Figure 4

Figure 5

Figure 6

In Figures 3 and 4, the impulse is the monetary base. There are four maturities from one month to 12 months, and six variables for each maturity term. We observe an increase in IP, CU, and PI for both types of utility functions. These effects do not persist for long and end within a year. CRRA-LBs do not respond greatly to an expansionary monetary base, whereas HABIT-LBs display slight declines. The magnitudes of the effects, however, are not large. Figures 5 and 6 show impulse responses to a shock to the BOJ's reserves. In contrast to the monetary base, we do not observe increases in IP, CU and PI for either type of utility functions. However, some of LBs respond slightly to an expansionary reserves policy. In summary, expansion of the monetary base as a method of quantitative easing policy is more effective in stimulating the real economy than expansion of the BOJ's reserves. However, the effects are not persistent. The results also suggest that neither the expansion of the monetary base nor the reserves have a great effect on the lower bounds of interest rates. When the BOJ adopted a quantitative easing policy in 2001, it targeted the BOJ's reserves. It has gradually increased the targeted amount of BOJ's reserves from ¥5 trillion to ¥35 trillion in January. 2004. When the BOJ implemented a quantitative and qualitative monetary easing (QQME) in April. 2013, it changed its target om the BOJ's reserves to the monetary base. According to both the BOJ and textbooks, the monetary base consists of the BOJ's reserves and the currency. In other words, the difference between the monetary base and the BOJ's reserves is the currency. However, during the period in which the quantitative easing policy was in place, the ratio of the monetary base to the BOJ's reserves varied significantly. As Figure 7 shows, this ratio declined in the midst of the 2000s and during the period in which the BOJ started a large-scale asset purchasing in the late 2000s. Recently, this ratio has become less than two for the first time. For this reason, a policy of monetary base targeting is different from that of targeting BOJ's reserves. The question arises as to which instrument a central bank should use as a quantitative measure. Our results favor the expansion of the monetary base policy to the expansion of the BOJ's reserves. However, neither of these two simple policies can lower the lower bounds of interest rates.

Figure 7

3.3.2. Granger-causality tests

In this subsection, we examine a Granger-causality test from the monetary pol- icy variable to the lower bound. Table 3 shows that the monetary base does not. Granger-cause the lower bound for each model and maturity term at the 5% significance level, except for the six- month HABIT. On the other hand, the BOJ's reserves Granger-cause the lower bound for less than six months under both utility types at the 5% significance level. The effect of an expansion of the BOJ's reserve policy on the lower bound occurs only in the shorter months. The BOJ used reserves as the quantitative easing measure from March 2001 to February 2006 and used the monetary base as such a measure from April 2013 [11]. As a robustness check, we examine the effects of the two measures used as the policy target, i.e., only during the period in which the measure was employed. To this end, we create the cross-term of the target and the period dummy that takes the value of one when the target is set as the quantitative easing measure. Table 4 presents the results of Granger-causality for this analysis. The monetary base did not Granger-cause lower bounds for any maturity term. The BOJ's reserve

[11] Our sample period ends in December 2016.

policy Granger-caused the lower bound only for a maturity term less than six-month under the Habit utility. Hence, the results in Table 3 are robust to restricting the appropriate period to that in which the BOJ's reserves are targeted as the measure of quantitative easing.

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Table 3

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Table 4

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3.3.3. Variance Decomposition

The final analysis examines a forecast error variance decomposition (FEVD) with a 5-year-horizon after the FAVAR estimation. As shown in Table 5, the FEVDs are mostly less than 10%. However, we have a relatively high FEVDs of the monetary base under Habit utility. This result suggests that a monetary base policy may affect the lower bounds of interest rates.

4. Conclusion

This study investigates the influence of an unconventional monetary policy, i.e., quantitative easing, on the economic activity and the lower bounds of interest rates. We estimate the shadow rates implied by consumption-Euler equations and calculate the time-varying lower bounds of interest rates implied by Black's option-based theory of shadow rate, employing AFNS term structure models. Estimation of the three-factor FAVAR system, including lower bounds of interest rates and quantitative easing measures, reveals that the monetary policy shocks have little effect on the lower bounds of interest rates. In particular, the effect of the monetary base is weaker than that of BOJ reserves. However, quantitative easing measures have real effects on the economy. The monetary base targeting policy is more effective than a BOJ reserves policy in stimulating economic activity. We observed the declines in lower bounds of interest rates in two periods, namely, the midst of the 2000s and 2010s. The former period corresponds to the BOJ's commitment period during which the BOJ announced its commitment to a quantitative easing policy until the Japanese economy overcame the deflation. The latter period corresponds to the period during which the BOJ employed large-scale asset purchasing, including a QQME policy. Our evidence suggests that a central bank can affect the lower bounds of interest rates by using appropriate measures. It also suggests that there was a room to lower the lower bounds of interest rates at the early stage of the deflation period in Japan. Our estimated shadow rates, which are calculated using consumption-Euler equations, are higher than those reported in the existing literature. These shadow rates that are consistent with Euler equations can be regarded as true shadow rates, because they satisfy individuals' optimality condition for consumption, as long as individuals freely buy and sell shadow bonds. Our analysis is limited in several respects. First, our analysis only considered a maturity term of less than one year. In principle, the methodology can be extended to a longer maturity, at the cost of reducing the sample size. For example, had we considered two years maturity, we would have lost 12 months of observations. Second, we only considered two types of utility functions.

The methodology can be extended to include a variety of utility functions as in Canzoneri et al. (2007). Third, we only considered AFNS(3) models as term structure models. The methodology can be extended to include a variety of term structure models, in particular, a broader class of affine term structure and squared-root process models. Fourth, we only considered a simple option pricing formula for the shadow bond. The methodology can be extended to include, for example, a model allowing for a time-varying strike price Such extensions are left for the future research.

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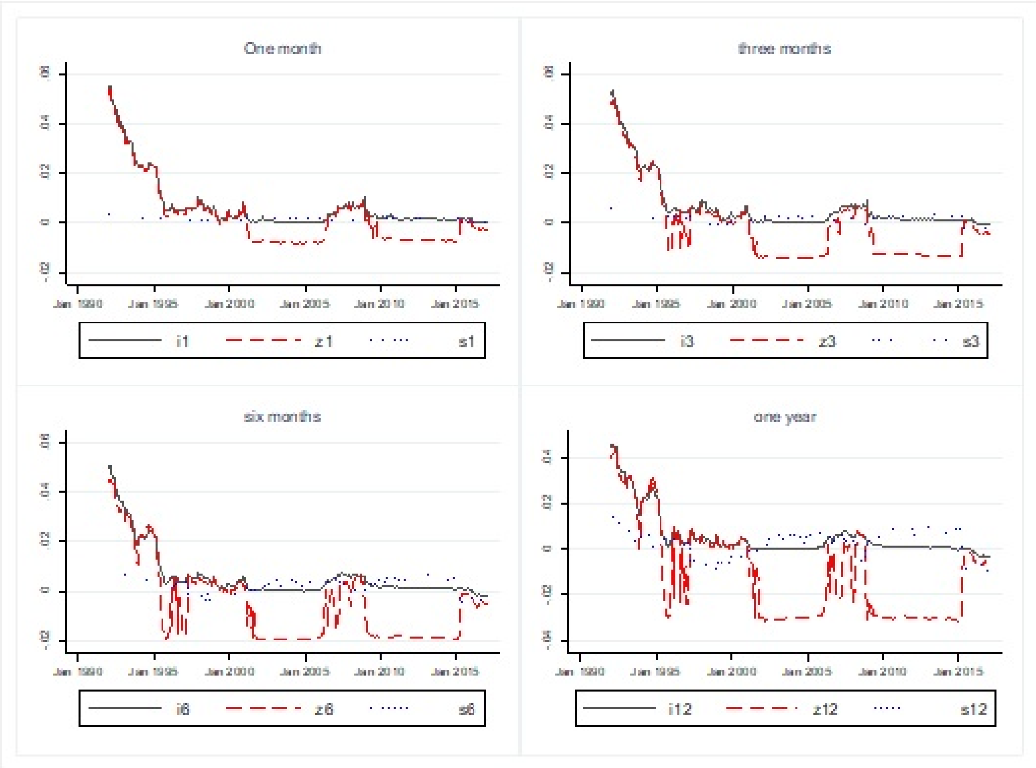
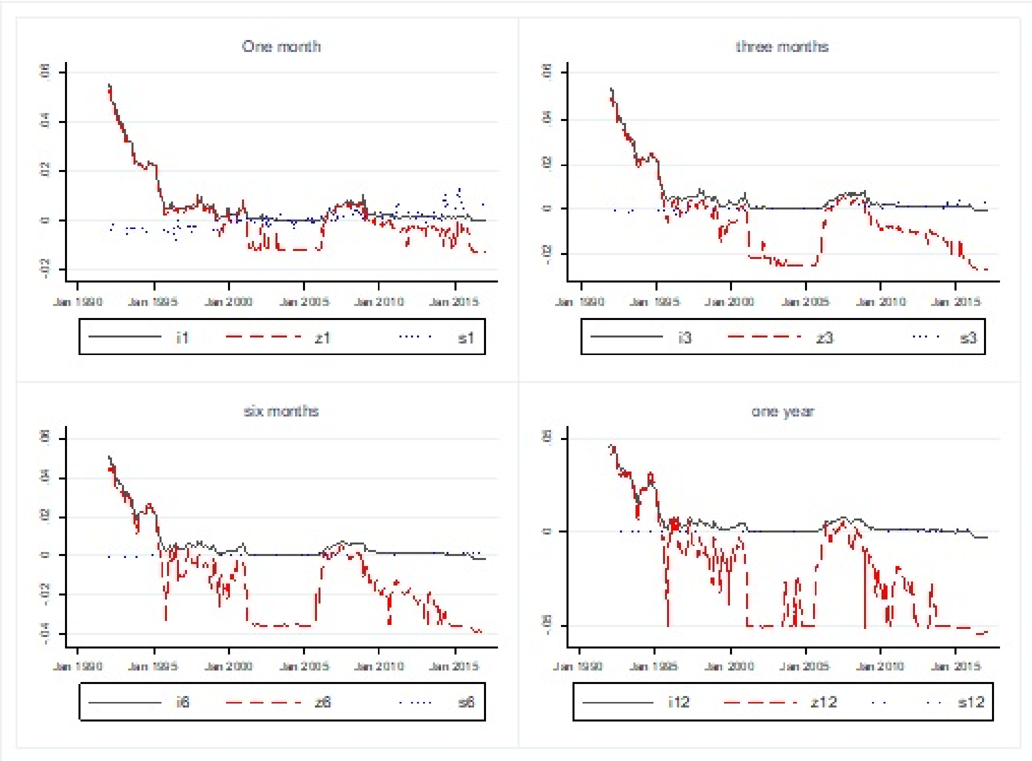
Figure 1: Estimated lower bound, shadow rate, and actual rate (CRRA, AFNS3)



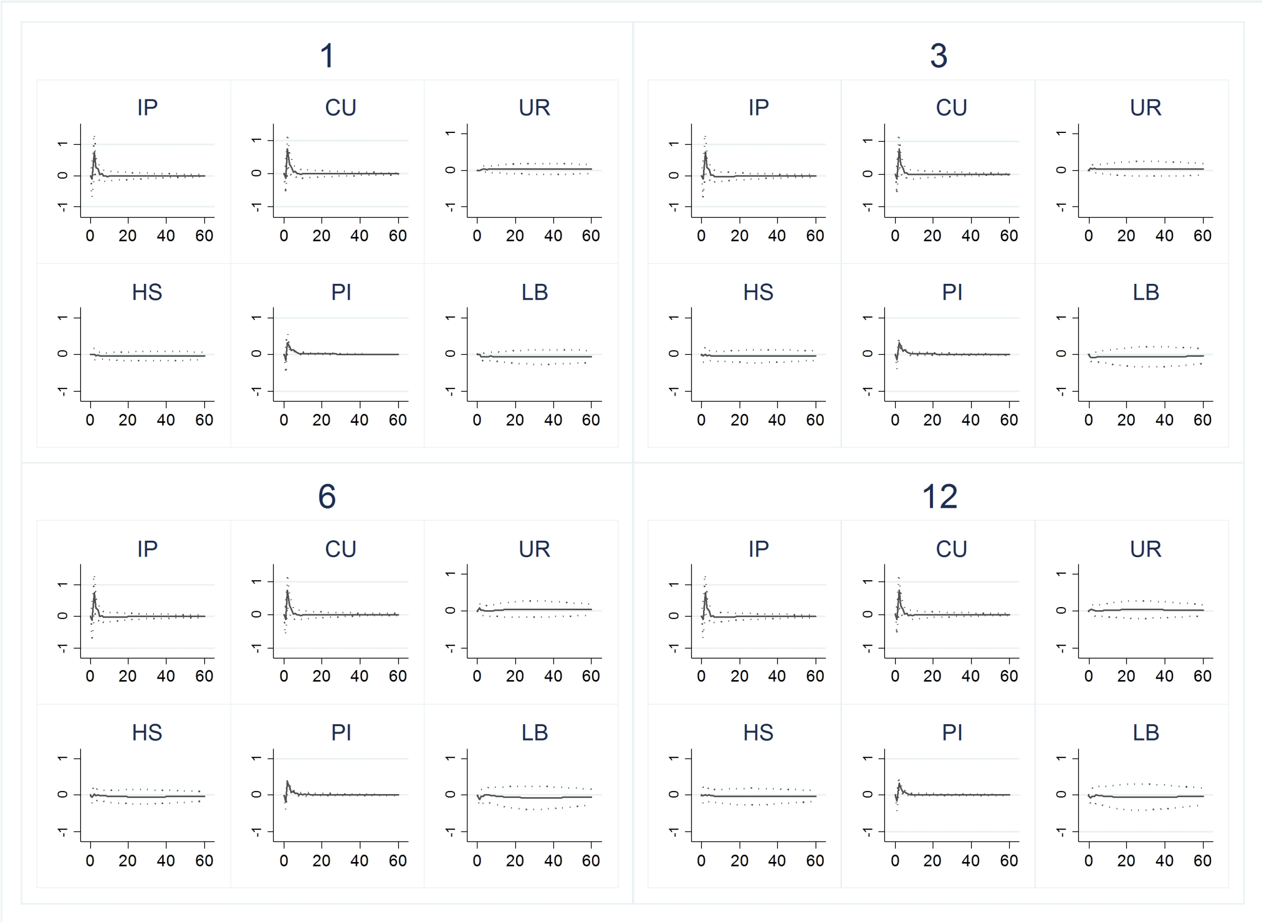
(Note) The figures indicate the actual rate i, the shadow rate s, and the lower bound z for one-, three-, six-, twelve-months.

Figure 2: Estimated lower bound, shadow rate, and actual rate (HABIT, AFNS3)



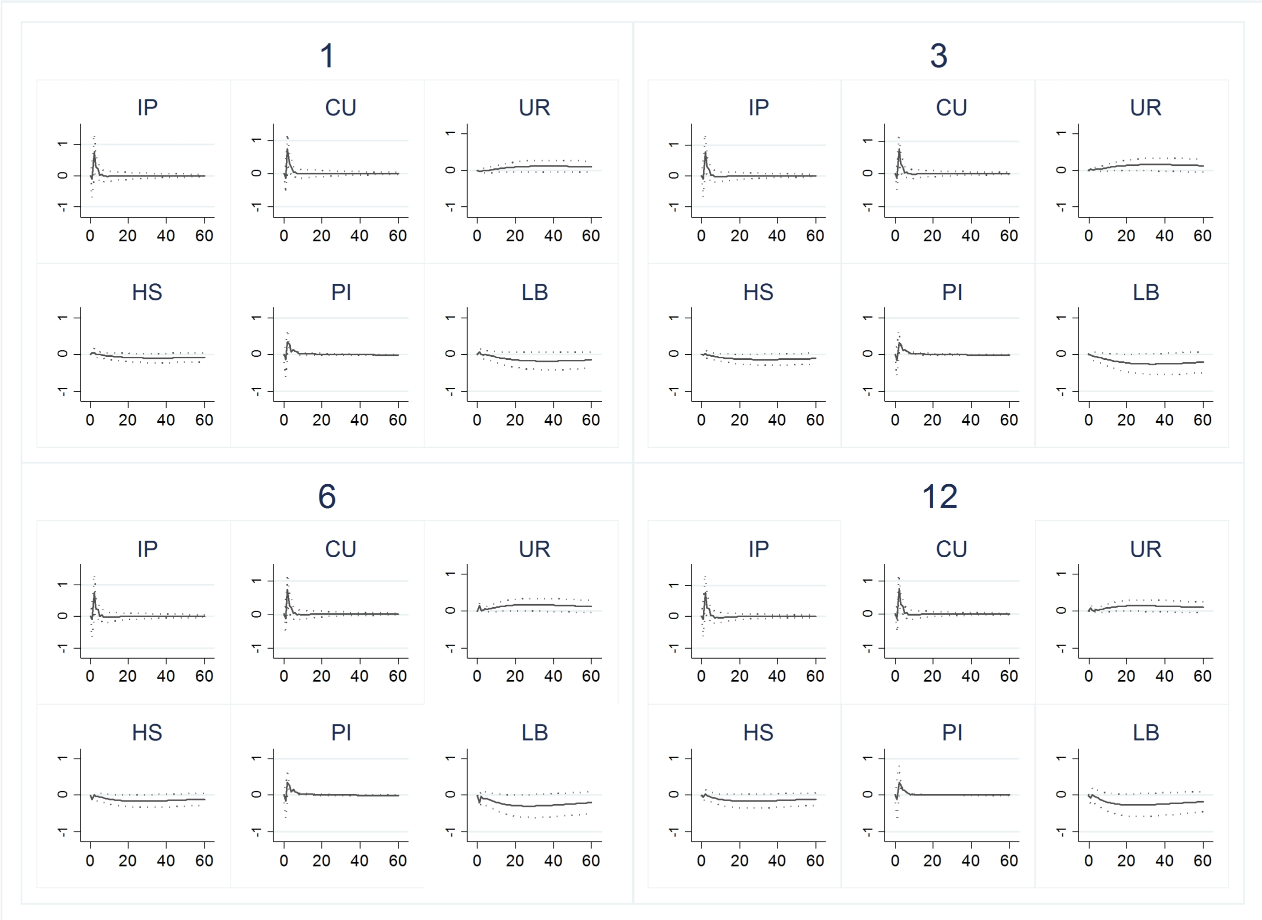
(Note) The figures indicate the actual rate i, the shadow rate s, and the lower bound z for one-, three-, six-, twelve-months.

Figure 3: Impulse response: Monetary Base, CRRA



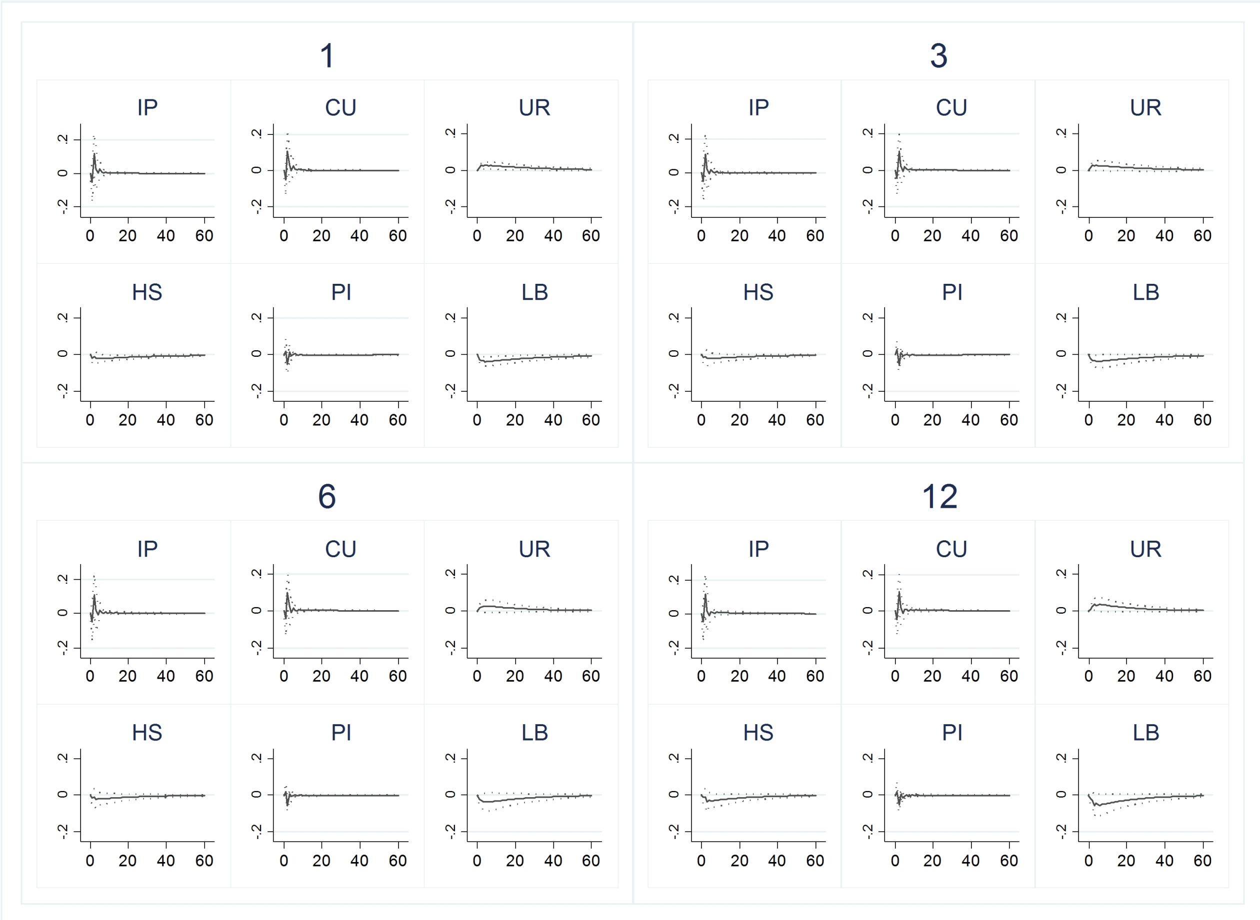
(Note) Impulse responses are generated from FAVAR with three factors, lower bound, and monetary policy variable. The dotted lines show 95% confidence intervals on both sides.

Figure 4: Impulse response: Monetary Base, HABIT



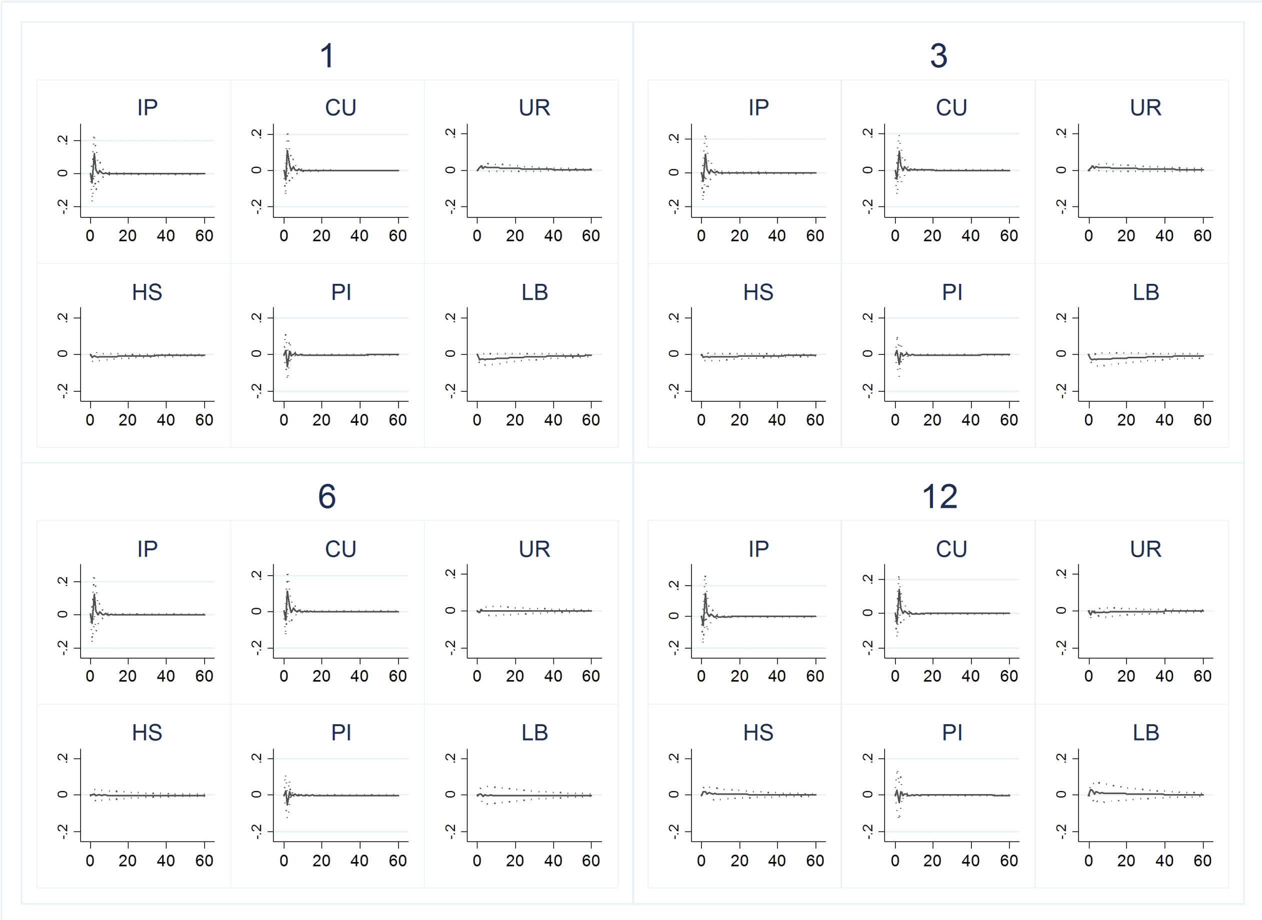
(Note) Impulse responses are generated from FAVAR with three factors, lower bound, and monetary policy variable. The dotted lines show 95% confidence intervals on both sides.

Figure 5: Impulse response: BOJ Reserves, CRRA



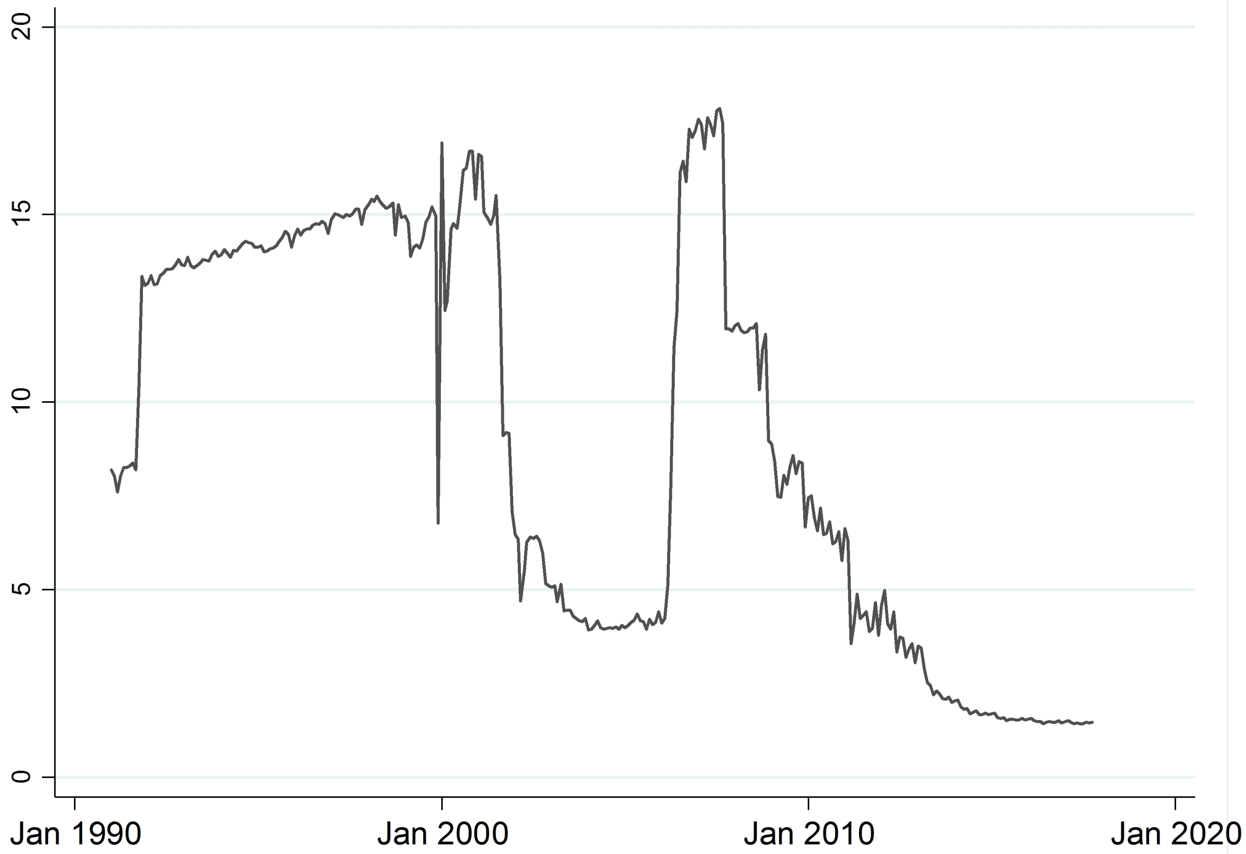
(Note) Impulse responses are generated from FAVAR with three factors, lower bound, and monetary policy variable. The dotted lines show 95% confidence intervals on both sides.

Figure 6: Impulse response: BOJ Reserves, HABIT



(Note) Impulse responses are generated from FAVAR with three factors, lower bound, and monetary policy variable. The dotted lines show 95% confidence intervals on both sides.

Figure 7: The ratio of monetary base to BOJ reserves



(Note) The graph displays the ratio of monetary base to BOJ reserves.

Table 1: Summary statistics

	One-month				Three-Month			
CRRA	Actual	Lower bound	Shadow	Gap	Actual	Lower bound	Shadow	Gap
1992–2008	0.927	0.589	0.170	0.338	0.888	0.244	0.193	0.644
2008–2017	0.203	-0.354	0.115	0.556	0.170	-0.803	0.134	0.973
All	0.649	0.226	0.149	0.422	0.612	-0.159	0.171	0.771
HABIT								
1992–2008	0.927	0.463	-0.122	0.464	0.888	-0.112	0.010	1.001
2008–2017	0.203	-0.415	0.395	0.617	0.170	-1.293	0.179	1.463
All	0.649	0.125	0.077	0.523	0.612	-0.566	0.075	1.178

	Six-Month				One-year			
CRRA	Actual	Lower bound	Shadow	Gap	Actual	Lower bound	Shadow	Gap
1992–2008	0.847	-0.112	0.221	0.959	0.822	-0.456	0.259	1.277
2008–2017	0.130	-1.245	0.167	1.375	0.081	-2.027	0.235	2.108
All	0.571	-0.548	0.200	1.119	0.537	-1.060	0.249	1.597
HABIT								
1992–2008	0.847	-0.751	0.040	1.597	0.822	-1.172	0.054	1.994
2008–2017	0.130	-2.424	0.124	2.554	0.081	-3.848	0.096	3.929
All	0.571	-1.394	0.072	1.965	0.537	-2.202	0.070	2.738

(Note) This table reports the summary statistics of actual rate (%), the estimated lower bound, and the nominal shadow rate implied by consumption-Euler equation, for each maturity and each utility function. The gap is actual rate minus lower bound.

Table 2: Estimation resuts of AFNS(3) models

Model	CRRA		Habit	
λ	1.238	***	0.040	***
	(0.013)		(0.005)	
κ^P_{11}	0.003	***	0.070	***
	(0.000)		(0.015)	
κ^P_{12}	-0.001	***	0.144	***
	(0.000)		(0.030)	
κ^P_{13}	0.272	***	-0.091	***
	(0.000)		(0.021)	
κ^P_{21}	0.402	***	0.238	***
	(0.000)		(0.057)	
κ^P_{22}	0.422	***	0.475	*
	(0.001)		(0.266)	
κ^P_{23}	-0.231	***	0.829	***
	(0.000)		(0.107)	
κ^P_{31}	0.026	***	0.130	
	(0.005)		(0.219)	
κ^P_{32}	0.017	***	0.115	
	(0.008)		(0.258)	
κ^P_{33}	0.648	***	0.515	
	(0.001)		(0.384)	
σ_{11}	0.009	***	0.008	***
	(0.000)		(0.002)	
σ_{22}	0.009	***	0.017	***
	(0.000)		(0.002)	
σ_{33}	0.011	***	0.076	***
	(0.001)		(0.017)	
σ_{21}	-0.989	***	0.750	***
	(0.000)		(0.267)	
σ_{31}	-0.254	***	0.088	
	(0.000)		(0.816)	
σ_{32}	0.395	***	0.180	
	(0.000)		(0.499)	
Log-likelihood	-37628.9		-27446.6	

(Notes) λ is a drift coefficient and σ_{ij} is an element of volatility matrix. κ^Ps are the drift coefficients under P-measure.

Table 3: Granger tests from instruments to lower bound

		Monetary base		BOJ reserve	
	Maturity	χ^2	p	χ^2	p
CRRA	1	0.441	0.802	15.959	0.000
	3	1.970	0.373	6.405	0.041
	6	1.992	0.369	2.610	0.271
	12	0.458	0.795	1.194	0.550
HABIT	1	4.497	0.106	8.059	0.018
	3	3.796	0.150	7.915	0.019
	6	9.050	0.011	0.347	0.841
	12	2.560	0.278	3.252	0.197

(Note) This table reports the results of Granger-causality test on the quantitative easing measure: Monetary base and BOJ reserves, for each month and for each utility function.

Table 4: Granger tests from instruments to lower bound with target period dummy

		Monetary base		BOJ reserve	
	Maturity	χ^2	p	χ^2	p
CRRA	1	0.607	0.895	0.885	0.829
	3	1.517	0.678	0.101	0.992
	6	1.141	0.767	0.122	0.989
	12	0.713	0.870	0.263	0.967
HABIT	1	1.303	0.728	16.950	0.001
	3	1.484	0.686	8.894	0.031
	6	5.575	0.134	0.523	0.914
	12	3.137	0.371	0.180	0.981

(Note) This table reports the results of Granger-causality test on the quantitative easing measure: Monetary base and BOJ reserves, for each month and for each utility function. To examine the effect of each measure (Monetary base or BOJ reserves) when each measure was targeted by BOJ, we use the cross-term of each measure and the targeted period dummy. The monetary-base-targeted-period-dummy takes one when monetary base was targeted and zero otherwise. The BOJ-reserves-targeted-period-dummy takes one when BOJ reserves was targeted and zero otherwise.

Table 5: Forecast error variance decomposition

	One-Month	Three-month	Six-Month	One-year
Monetary Base				
CRRA	0.035	0.017	0.013	0.006
HABIT	0.145	0.222	0.209	0.149
BOJ reserves				
CRRA	0.119	0.054	0.033	0.047
HABIT	0.039	0.032	0.000	0.005

(Note)This table indicates the results of the variance decomposition at five-year-horizon.